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VELOCITY RECOVERY FACTORS OF A PARTICLE REPELLED FROM A SOLID SURFACE

A. L. Stasenko UDC 556.6.071.6:532.529

On the basis of the published experimental data, recovery factors of velocity components of microparticles impacting a target heuristic expression, which are suitable for a wider class of pairs of materials of colliding bodies, have been proposed. The considered range of values of impact velocities is bounded on the side of both low velocities (since the adhesive forces are neglected) and higher velocities (at which irreversible deformation of the particle and/or the target occurs).

Introduction. In the flow of bodies in a high-velocity two-phase stream, an important role is played by the physics of the microparticle–surface collision. In particular, the structure of the compressed layer strongly depends on the rebound velocity components of the particle. It is natural to suppose that the result of an impact depends on such physicochemical properties of substances as their density, the Young and shear moduli, their related Poisson coefficient, the yield point, the surface density of adhesion, and the coefficient of sliding friction.

The present paper considers the range of impact velocities at which complete plastic deformation of the target or the particle excluding the rebound of the latter does not yet occur. On the other hand, the too small impact velocities below which "adhesion" of particles to the surface occurs have not been considered either. These limitations correspond to the conditions of those "reference" experiments on whose results the proposed physical interpolation is based.

Main Results of the "Reference" Experiments. Experimental studies of the rebound of solid particles from solid surfaces at impact velocities that are of interest for the present work were begun long ago. As an example, let us compare two groups of experiments performed in different countries, at different times, and with different substances. Figure 1 presents the experimental data on the recovery factors of the normal $a_n = v_{2n}/v_{1n}$ and tangential $a_\tau = v_{2\tau}/v_{1\tau}$ components of the particle. Figure 1a and b corresponds to the case of light ($\rho_p \sim 1.1$ g/cm^3) and soft balls (apparently because of polystyrene) impacting "hard" steel [1]. Figures 1c and d presents the results of [2] where particles of electrocorundum (substance second only to diamond in hardness) impacting "softer" bodies (steel, copper, lead) were used. In so doing, the electrocorundum particles bore a weak resemblance to globular ones: they represented "sharp-grained prolate fragments" whose size could only be characterized by some equivalent diameter.

In the first group of experiments, fairly large particles of diameter 300–2000 μm were used, and it was noted that close results had been also obtained in [3] for smaller particles of size 15–300 μm. In the second group of experiments [2], the characteristic diameter of particles was 20–100 μm.

The figures presented gave an indication of the scatter of the measurement data obtained in each group of experiments. At the same time it is seen that there is a striking agreement between the angular dependences $a_n(\beta_1)$ obtained for so widely differing values of the coefficients of elasticity and density of particles-projectiles and bodies-targets. It was no accident that the experimentalists of [1, 2] came to the single conclusion: "the main parameter influencing the coefficients a_n and a_τ is the angle of incidence of particles on the surface." However, the data presented in Fig. 1 point to the validity of this conclusion only for a_n. The values of a_τ evidently depend on the properties of the materials of colliding bodies. Unfortunately, the authors of the above-mentioned works give the interpolation dependences of their measurements only for particular pairs of projectile and target materials, which, generally speaking, are not suitable for other materials. The same can also be said about work [4] where a thorough interpolation of the results of [2] was carried out for each pair separately.

Prof. N. E. Zhukovskii Central Aerodynamic Institute, 1 Zhukovskii Str., 140180, Zhukovskii, Moscow Region, Russia; email: stasenko@serpantin.ru. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 80, No. 5, pp. 38–44, September–October, 2007. Original article submitted March 20, 2006.

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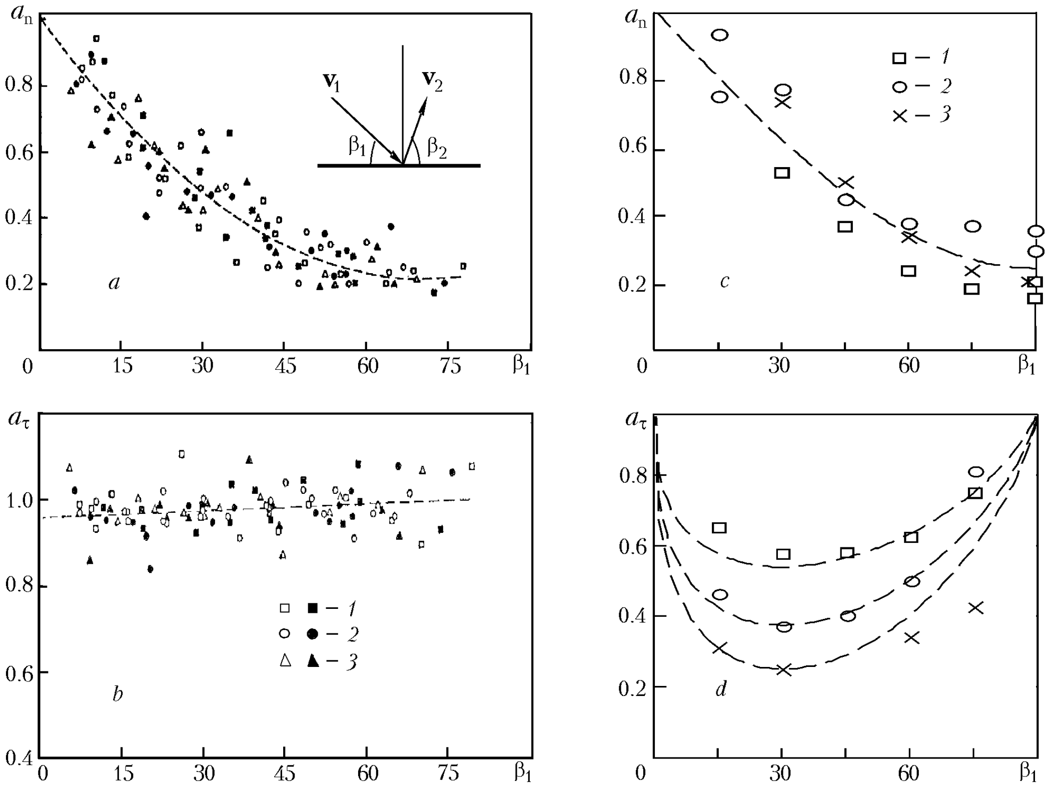

Fig. 1. Experimental measurement data on the recovery factors of the velocity components of particles impacting a surface: a, b) light particles and the surface of a steel blade [1]; open marks denote the suction face, solid ones the higher-pressure face [particle diameters: 1) 300, 2) 1000, 3) 2000 μm]; c, d) electrocorundum particles and wedge surface from different metals [2] [1) steel, 2) copper, 3) lead].

In this connection, in the present paper we propose a simple heuristic theory which, basing on the given experimental data, takes into account the dependences of a_n and a_τ not only on the sliding angle of particles, but also on the properties of the materials of the particle and the body being flown.

Search for Relation between the Experimental Results and the Physicomechanical Properties of the Colliding Bodies. From Fig. 2 [2] pertaining to the case of normal collision ($\beta_1 = \pi/2$) it is seen that at a certain velocity of incidence of a particle v_{1n}^* its rebound does not occur. This effect can be associated with the yield point of the particle (index p) and/or the surface on which the particle is incident (index s). Suppose that at this limiting velocity the whole of the kinetic energy of the particle is expended in the work of deformation of the collision participants. The force crushing the particle has the order $\sigma_p \pi a_p^2$ and its work is proportional to $\sigma_p \pi a_p^2 2a_p$. Suppose then that the characteristic size of the region of plastic deformation of the body is proportional to the diameter of the incident particle: $2a_s \sim 2a_p$. Let us write the condition of energy conservation in the form of the "law of parallel conductivities"

$$\frac{1}{m_p v_{1n}^{*2}/2} = \frac{1}{\xi_p \sigma_p \pi a_p^2 2a_p} + \frac{1}{\xi_s \sigma_s \pi a_p^2 2a_p}, \tag{1}$$

where ξ_p and ξ_s are fitting dimensionless coefficients. The point of such writing is obvious: if at least one of the materials is absolutely plastic (σ_p or $\sigma_s \to \infty$), then the particle does not rebound even at an impact velocity far from the conditions of plastic deformation of the other material ($v_{1n}^* \to 0$); but if one of the materials is absolutely rigid (σ_p or $\sigma_s \to \infty$), then this limiting velocity is determined by the properties of the other material.

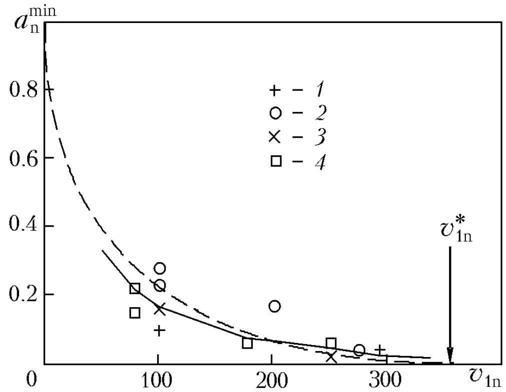

Fig. 2. Influence of the velocity of incidence of particles on a plate set across the flow on the velocity recovery factor (plate material — steel; $\beta_1 = 90^o$) at a particle diameter: 1) 23; 2) 32; 3) 88; 4) 109 μm; dashed line — proposed interpolation.

TABLE 1. Physical Properties of Interacting Bodies

Material	ρ, 10^3 kg/m^3	σ, 10^7 N/m^2	$(c_{\parallel}c_{\perp})^{1/2}$, m/sec	μ	β_1^{min}, deg	α_τ^{min}
Electrocorundum	4	74	8800	—	—	—
Steel	7.8	23	5550	0.3	28	0.54
Copper	8.9	6.85	3260	0.36	31	0.375
Lead	11.3	0.5—1	1230	0.45	33	0.25
Polystyrene	1.1	3.9	1620	—	—	~1

Note. Lines in the table point to the absence of data, which is unimportant for the present work.

Taking into account that $m_p = \frac{4}{3}\pi\rho_p a_p^3$, from the latter equality we get

$$v_{1n}^* = \left[\frac{3}{\rho_p}\left(\frac{1}{\xi_p\sigma_p} + \frac{1}{\xi_s\sigma_s}\right)^{-1}\right]^{1/2} .$$

For the pair electrocorundum–steel we have (see Table 1, whose data were taken from reference books [5, 6]): $\rho_p = 4 \cdot 10^3$ kg/m^3, $\sigma_p = 74 \cdot 10^7$ N/m^2, $\sigma_s = 23 \cdot 10^7$ N/m^2, whence

$$v_{1n}^* = \left[\frac{3 \cdot 10^7}{4 \cdot 10^3}\left(\frac{1}{74} + \frac{1}{23}\right)^{-1}\right]^{1/2} = 360 \text{ m/sec} .$$

This value is marked by an arrow in Fig. 2. It is seen that the proposed simple estimation is well confirmed by the experiment (at assumed values of ξ_p, $\xi_s = 1$).

Let us estimate above what temperature the colliding bodies (electrocorundum–steel) can be heated upon an absolutely inelastic collision at the obtained limiting velocity:

$$\Delta T \le \frac{v_{1n}^{*2}}{2}\frac{1}{C_p + C_s} = \frac{360^2}{2\,(500 + 1000)} \approx 40 \text{ K}$$

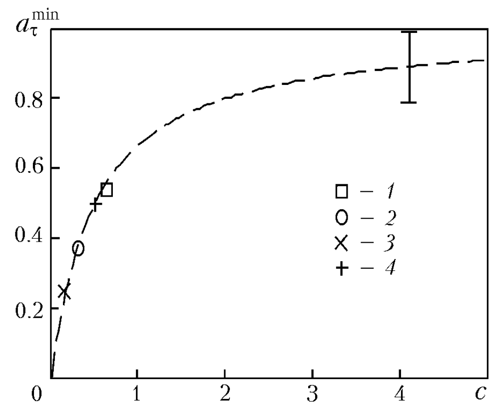

Fig. 3. Dependence of the least value of the recovery factor of the tangential component of the particle momentum on the properties of the material of the particles and bombarded surface (dots show the experiments of [2]); impacts of electrocorundum particles: 1) on steel; 2) on copper; 3) on lead; 4) on glass; the vertical segment corresponds to the experiments [1] with light balls bombarding steel; the dashed line shows the proposed interpolation.

(ΔT is the temperature gradient, K; C_p and C_s are the specific heat capacities, J/(kg·K)). Consequently, even at such an impact velocity neither the particle-projectile nor the body-target are heated to the melting temperature.

Let us assume that at low velocities of normal collision an absolutely elastic rebound occurs, i.e., $a_n = 1$. Then the dependence of a_n on the velocity given in Fig. 2 can be interpolated, e.g., by the curve

$$a_n\left(\frac{\pi}{2}\right) = \left[1 - \left(\frac{v_{1n}}{v_{1n}^*}\right)^q\right]^p .$$

It will be recalled that v_{1n}^* is the value of the normal velocity component at which the particle does not rebound anymore. The exponents q and p are selected from comparison with the experimental data.

In Fig. 2, the dashed line corresponds to the values of $1/q = p = 2$. The value of $a_n = 1$ at a zero impact velocity points to the fact that the deformations are insignificant and the impact is absolutely elastic (with the adhesion forces neglected, see further remark 4).

Let us consider the recovery factor of the tangential velocity component a_τ (see Fig. 1b, d). It is seen that as the relative rigidity of the projectile/target material decreases, the value of a_τ monotonically tends to unity. The angular dependence has a maximum at $\beta_1 \approx 30^o$. In [2], it was noted that when particles meet the surface of a wedge, their trajectories deviate by 2–3^o from the initial ones (taking place "at infinity") because of the gas flow curvature; however, this difference is immaterial against the background of the spread of the experimental results.

From the physical considerations it is clear that at a tangential impact not only the Young modulus E, but also the shear modulus G must play a role. In particular, they influence the values of the longitudinal and transverse components of the velocity of sound in the projectile and target materials $c_\parallel = (E/\rho)^{1/2}$, $c_\perp = (G/\rho)^{1/2}$.

In interpolating the experimental data presented, one can attempt to find the dependence of a_τ on different combinations of these velocities: the mean value ($c = 0.5(c_\parallel + c_\perp)$); the mean harmonic value ($2c^{-1} = c_\parallel^{-1} + c_\perp^{-1}$); and the "mean cubic harmonic" value ($3c^{-3} = c_\parallel^{-3} + 2c_\perp^{-3}$) which is used to count the number of oscillation modes of a crystal. Comparison of different interpolations has shown that the data of experiments best fall on one curve if the mean geometric value $c = (c_\parallel c_\perp)^{1/2}$ is used (Fig. 3). Then the sought value can be given as

$$a_\tau^{min} = \frac{c}{c + 1/2}, \quad c = \frac{c_s}{c_p} = \left[\frac{(c_\parallel c_\perp)_s}{(c_\parallel c_\perp)_p}\right]^{1/2} .$$

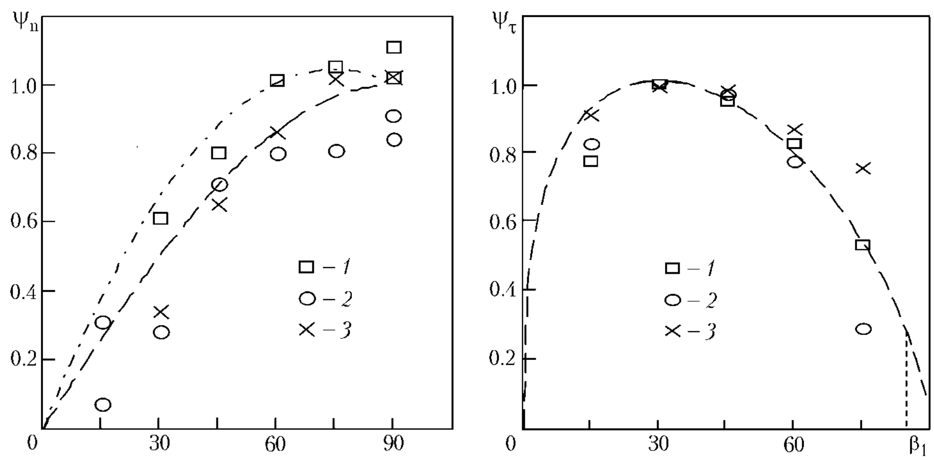

Fig. 4. Angular dependences of recovery factors of the normal and tangential components of particles (dots, see Fig. 1a, d). The dashed line corresponds to the proposed interpolation, the dash-dot line — to the interpolation [1] for light balls impacting steel.

Note that the least values of $a_\tau = a_\tau^{\min}$ are attained at a sliding angle whose value can be related to the Poisson modulus μ by the relation $\tan \beta_1^{\min} = \sqrt{\mu}$. The values of this angle are given in Table 1 and lie near $\beta_1^{\min} = 30^{\circ}$. Therefore, for simplicity, a single (μ-independent) function of the sliding angle was taken:

$$\frac{1 - a_\tau(\beta_1)}{1 - a_\tau^{\min}} = \left(\frac{\beta_1}{\pi/6}\right)^{1/3} \left(\frac{\pi/2 - \beta_1}{\pi/3}\right)^{2/3} = \psi_\tau(\beta_1)\,.$$

It gives a zero value of a_τ at the ends of the interval $0 \le \beta_1 \le \pi/2$ and a minimum value $a_\tau^{\min}$ at $\beta_1 = \pi/6$ (Fig. 4, dashed curve). In the case of the pair polystyrene balls–steel, this dependence "drowns" inside the spread of experimental data [1] in the vicinity of $a_\tau \sim 1$ (Figs. 1a, b and 3).

Proposed Expressions for the Recovery Factors of Particle Velocity Components. The following interpolations of the given experimental results are recommended:

$$\psi_n \equiv \frac{1 - a_n}{1 - a_n^{\min}} = \sin \beta_1\,, \quad \psi_\tau \equiv \frac{1 - a_\tau}{1 - a_\tau^{\min}} = \left(\frac{\beta_1}{30}\right)^{1/3} \left(\frac{90 - \beta_1}{60}\right)^{2/3},$$

where

$$a_n^{\min} \equiv a_n(90^{\circ}) = \left[1 - \left(\frac{v_{1n}}{v_{1n}^{*}}\right)^{1/2}\right]^2, \quad a_\tau^{\min} \equiv a_\tau(30^{\circ}) = \frac{c}{c + 1/2}\,.$$

These "universal" interpolations are shown by dashed curves in Figs. 2–4, where also the recalculated data of Fig. 1c and d are given. In Fig. 4, the dash-dot curve shows also the interpolation of the experimental work [1] in the form of $a_n = 1 - 0.02108\beta_1 + 1.417 \cdot 10^{-4}\beta_1^2$ (the sliding angle β_1 is given in degrees).

Remarks. 1. We have not considered the interaction of electrocorundum particles with a glass target that was also investigated in [2], since glass belongs to another class of substances. Therefore, the proposed interpolation does not pretend to be universally suitable for substances of all classes. In [2], this is given as follows: "In the series of investigated substances, only glass has a higher coefficient a_n. Evidently, this is explained by the fact that glass sharply differs from other materials in its physicomechanical properties — it is a hard, brittle material. However, the character of the angular dependence of the coefficient a_n is the same for all investigated materials."

2. Exact values of $a_\tau = 1$ obtained from the given expressions at $\beta_1 \to 0$ (sliding impact) and $\beta_1 \to \pi/2$ (normal impact) are physically unattainable for two reasons: first, because of the roughness and waviness of the surface of the body, which takes place everywhere in reality; second, because of the presence of friction (this is evidenced, in particular, by the values of $a_\tau > 1$ obtained in the experiment). The first of them leads to a spread of the angles of collision about the vertical to the "median surface." The second reason is the cut-off of the value of β_1 at which the tangential velocity component of particles disappears upon rebound. Let us explain the latter fact. The change in the normal and tangential components of the particle momentum results from the impact under the action of the normal and tangential components of the momentum, which can be written conventionally in the form $N\tau_{coll}$ and $k\ N\tau_{coll}$, where k is the friction coefficient and τ_{coll} is the characteristic time of collision:

$$m\,(v_{2n} - v_{1n}) = N\tau_{coll}\,, \quad m\,(v_{2\tau} - v_{1\tau}) = kN\tau_{coll}\,.$$

Taking into account that $v_{1n} = -v_1 \sin \beta_1$, $v_{2n} = a_n v_{1n}$, $v_{1\tau} = v_1 \cos \beta_1$, we get

$$\cot \beta_2 = (\cot \beta_1 - k\,(1 + a_n))/a_n\,.$$

It is seen that $\beta_1 \to \pi/2$ at a value of $\beta_1^* < \pi/2$ determined by the condition $\cot \beta_1^* = k(1 + a_n)$. Thus, dry friction completely "eats away" the tangential momentum of the particle inside the values of the angle of incidence (counted off from the vertical):

$$0 \le \alpha_1 < \alpha_1^* = \pi/2 - \beta_1 = \arctan\,[k\,(1 + a_n)]\,.$$

For example, at $k = 0.1$ we obtain $a_1^* \ge 6^o$. Inside this interval $a_\tau = 0$ (Fig. 4, dotted vertical).

3. We have not considered here the swirl of particles upon collision, since it is difficult to describe the rotation of rebounded electrocorundum particles — "sharp-grained prolate fragments" [2]. For the model of balls, the swirling of particles and the influence of roughness were considered in [7].

4. For completeness of the review of the experimental data, we note a group of experimental-theoretical works in which relatively small particle-surface impact velocities were considered. In [8], steel microspheres of diameter 10–125 μm and lycopodium spores (bioaerosol) impacting in a vacuum, a molecular-smooth silicon crystal (up to 5 Å) with velocities not exceeding 4.4 m/sec was used. In [9], glass balls (of diameter 100–500 μm) and quartz particles (~100 μm) were entrained by a horizontal air flow in a channel of rectangular section 30 × 300 mm^2. Particular consideration was given to the scattering of particles by the very rough (up to 25 μm) steel surface of the channel walls. In [10], measurements of the kinetic energy loss were made for polystyrene particles of diameter from 3 to 7 μm) accelerated by an air flow and impacting polished surfaces of molybdenum, silicon, mica, and a fluorine-carbon polymer. Only normal incidence was investigated. It was shown that with decreasing impact velocity, upon reaching a certain critical value (~20 m/sec), the increase in the coefficient a_n is replaced by its decrease, and the dependence on both the target material and the particle size is revealed therewith. These works can be useful subsequently in taking into account the adhesion forces, the role of the roughness of the surface being bombarded, and the shape of incident particles.

Conclusions. Analytical expressions for the recovery factor of the normal and tangential components of the velocity of a spherical particle rebounding from a solid surface have been proposed. They are based on the interpolation of the experimental data obtained by different researchers and covering a wide range of particle sizes and physical properties of materials, velocities, and angles of incidence. The generalization is in that we related the experimental data to such characteristics as the yield point and the transverse and longitudinal velocities of sound (depending on the density of colliding bodies and their shear and Young moduli). The proposed expressions do not pretend to be unique, since an infinite number of analytical curves can be laid on the field of experimental points (the more so, as the particles used in the experiments were not always spherical). However, "relating" to the physical properties of the solid permits hoping that the formulas proposed will also be useful for other pairs of substances of particles and the surface bombarded by them.

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NOTATION

a_n and a_τ, recovery factors of the normal and tangential components of the particle velocity; a_p, particle radius, μm; $c_\parallel$ and $c_\perp$, velocities of the transverse and longitudinal elastic waves, m/sec; E and G, Young and shear moduli, N/m^2; m_p, particle mass, kg; $\mathbf{V}_1(v_{1n}, v_{1\tau})$ and $\mathbf{V}_2(v_{2n}, v_{2\tau})$, velocities of incident and repelled particles and their components, m/sec; β_1 and β_2, sliding angles of incident and repelled particles, deg; μ, Poisson coefficient; ρ_p and ρ_s, particle and target densities, kg/m^3; σ, yield point, N/m^2. Subscripts: 1, 2, parameters of incident and repelled particles; coll, collision; p, s, parameters of the particle and target materials; n and τ, normal and tangential components; min, minimal; *, threshold value.

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